Efficient Computation of the Periodic Green's Function in Layered Dielectric Media

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Abstract—**This paper presents a novel technique for the efficient computation of the *periodic* Green's function in layered dielectric media. The technique is based upon expanding the spectral Green's function into a finite number of inverse-transformable complex exponential functions. This enables the use of Poisson's summation formula to express the periodic Green's function as a combined sum of spectral terms and spatial terms each set of which is rapidly convergent. Numerical results are obtained for the "on-plane" case, in which the direct summation of the series converges extremely slowly. Using the accelerated summation formula of this paper, a computation time reduction of 160 fold is obtained. The proposed technique is useful as it can be applied to a wide class of problems where periodic structures are to be modeled.**

I. Introduction

The electromagnetic modeling of periodic structures in layered dielectric media involves the formation of an integral equation which has as its kernel a Green's function series that converges very slowly. In using the moment method to determine the radiation or scattering from a periodic array, repeated evaluations of the Green's function series are required to fill in the impedance matrix of the structure being modeled. The slow convergence of the series would, therefore, result in a considerable amount of computation time. Several papers have recently proposed series acceleration techniques to enhance the convergence of the periodic Green's function using the spectral- and spatial-formulations in conjunction with Kummar's, Poisson's, and Shank's transformations [1]–[6]. However, the techniques proposed in these papers have been applied only to the *free-space* periodic Green's function. It is therefore desirable to develop a technique which accelerates the convergence of the *layered media* periodic Green's function. The contribution of this paper is to provides such a technique which can apply to a wide class of problems involving general layered media structures.

The technique is a combination of two: the complex images [7], [8] for the layered media in the vertical (z) direction and the series accelerating technique of Singh *et al.* [3]. The combination is then improved in convergence using Poisson's summation formula. The complex image technique uses Prony's method [9] to approximate the spectral Green's function by a set of complex exponentials which are analytically inverse Fourier-transformable. The functional form of these spectral exponentials enables them to be transformed to the spatial domain in closed-form through the use of Sommerfeld identity [10]. The spatial solution is interpreted as the response due to the source plus a few complex images of complex amplitudes and locations [7], [8]. It is found that the asymptotic behavior of the spectral exponentials causes the slow convergence of the Green's function series. To overcome this problem, this paper develops a generalized form from the accelerating technique of the free-space periodic Green's function [3]. It starts by first subtracting out the asymptotic behavior of the spectral domain and then making use of Poisson's summation formula to add it back in the spatial domain. The efficient complex image technique [7], [8] is then used to express the added spatial contribution in a simple closed form. The final solution of the periodic Green's function would, therefore, consist of a combination of a spectral series and a spatial series, each of which is rapidly convergent. This makes the overall mixed spectral-spatial summation of the periodic Green's function also rapidly convergent.

As an application of the technique presented in this paper, numerical experiments have been conducted to compute the periodic Green's function of a two-dimensional array of point dipole sources printed on a microstrip substrate of arbitrary thickness. The results show the number of terms required for the mixed spectral-spatial summation to converge, for different values of the substrate thickness and relative permittivity.

II. Theory

In determining the radiation at an observation point (x, y) from a two-dimensional infinite array of *phase shifted* point sources located at (x', y') in each unit cell as shown in Fig. 1, evaluations of the different components of the *vector* and *scalar* potentials are needed. Each Green's function component of these potentials can be expressed in terms of a spectral sum with the general form

$$G_p = \frac{1}{ab} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \tilde{G}_{mn}(k_{xm}, k_{yn}) \cdot e^{-jk_{xm}(x-x')} \cdot e^{-jk_{yn}(y-y')} \tag{1}$$

with the discrete values of k_{xm} and k_{yn} given by

$$k_{xm} = k_x^i + \frac{2\pi m}{a}, \quad k_{yn} = k_y^i + \frac{2\pi n}{b} \tag{2}$$

a and b represent the x- and y-periodicities of the structure, as shown in Fig. 1. k_x^i and k_y^i are wavenumbers associated with

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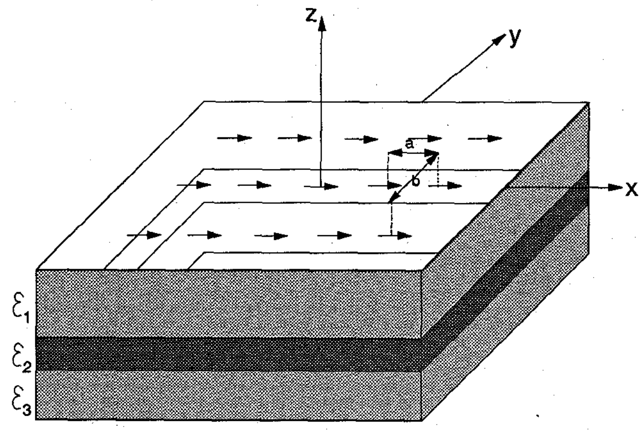

Fig. 1. Geometry of an infinite array of point dipole sources above a general layered media structure.

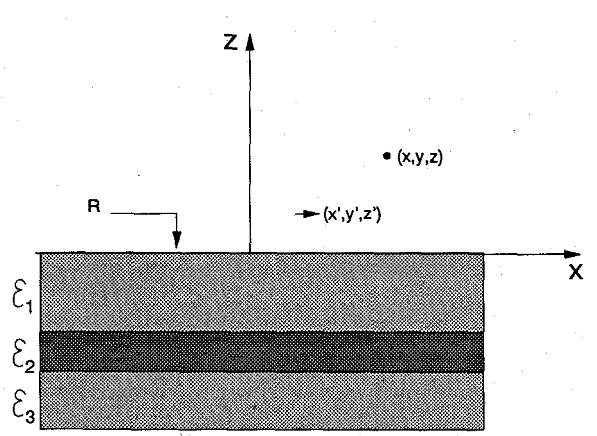

Fig. 2. A point dipole source above a layered media structure.

the phase shifted plane wave. They are given by

$$k_x^i = k_0 \sin\theta_i \cos\phi_i, \quad k_y^i = k_0 \sin\theta_i \sin\phi_i \tag{3}$$

where (θ_i, ϕ_i) are the spherical coordinate angles of an incoming or outgoing wave and k_0 is the free-space wavenumber. In (1) $\tilde{G}_{mn}$ represents the *spectral* Green's function of an infinitesimal dipole point source at (x', y', z') above a general layered media structure as shown in Fig. 2. It can be written in the following form of superposition of plane waves

$$\tilde{G}_{mn} = \frac{1}{j2k_{z0}}[e^{-jk_{z0}(z-z')} + \mathrm{R}e^{-jk_{z0}(z+z')}] \tag{4}$$

where

$$k_{z0} = \begin{Bmatrix} \sqrt{k_0^2 - (k_{xm}^2 + k_{yn}^2)}, & k_0^2 \geq k_{xm}^2 + k_{yn}^2 \\ -j\sqrt{(k_{xm}^2 + k_{yn}^2) - k_0^2}, & k_0^2 < k_{xm}^2 + k_{yn}^2 \end{Bmatrix} \tag{5}$$

R in the above expression of $\tilde{G}_{mn}$ represents the spectral reflection coefficient due to the layered media, as shown in Fig. 2. Its expression can be obtained from the specific problem under consideration. For example, the Appendix has the expression of R for the microstrip substrate problem.

It is well known that the spectral series summation in (1) is slowly convergent, especially for the "on-plane" case where $z = z'$. To overcome the slow convergence problem, a series accelerating technique proposed by Singh *et al.* [3] for the free-space periodic Green's function is implemented. The success in generalizing such a technique to the layered media periodic Green's function lies in obtaining a closed-form expression for the spatial Green's function. This is accomplished using the theory of complex images presented as follows.

A. The Complex Images

We approximate the spectral reflection coefficient R by a short series of exponential functions [7], [8]:

$$R = \sum_{i=1}^{N} A_i e^{jB_i k_{z0}}, \qquad N \leq 5 \tag{6}$$

where A_i and B_i are complex coefficients obtained for a chosen N by the application of Prony's method [9]. The approximation in (6) of the spectral reflection coefficient R for different layered media structures was investigated here as well as in [7]. It was found that in all cases taking N in the range $3 \sim 5$ is sufficient to yield an accuracy of better than 0.1%. From (6) into (4), $\tilde{G}_{mn}$ becomes

$$\tilde{G}_{mn} = \frac{1}{j2k_{z0}}\left[e^{-jk_{z0}(z-z')} + \sum_{i=1}^{N} A_i e^{-jk_{z0}(z+z'-B_i)}\right]. \tag{7}$$

The advantage of the short series approximation of R as given in (6) becomes now evident as the inverse Fourier-transformation of $\tilde{G}_{mn}$ in (7) yields a closed-form spatial expression for G_{mn}:

$$G_{mn} = \frac{e^{-jk_0 r_0}}{4\pi r_0} + \sum_{i=1}^{N} A_i \frac{e^{-jk_0 r_i}}{4\pi r_i} \tag{8}$$

where $r_0 = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$ and $r_i = \sqrt{(x-x')^2 + (y-y')^2 + (z+z'-B_i)^2}$. G_{mn} is interpreted as the response due to the source and a set of "N" complex images of complex amplitudes (A_i's) and complex locations $(-z' + B_i)$.

B. The Series Accelerating Technique of Singh et al. [3]

The rate of convergence of the Green's function series G_p in (1) is governed by the asymptotic behavior of $\tilde{G}_{mn}$. To derive the expression of this asymptotic behavior we introduce an attenuation constant u such that

$$\begin{aligned} k_{z0} &= -j\sqrt{k_{xm}^2 + k_{yn}^2 - k_0^2} \\ &= -j\sqrt{(k_{xm}^2 + k_{yn}^2 + u^2) - (k_0^2 + u^2)}. \end{aligned} \tag{9}$$

Clearly from (2) as $(m, n) \to \infty$ both $(k_{xm}, k_{yn}) \to \infty$. Therefore, we can take $k_{z0} \to -j\sqrt{k_{xm}^2 + k_{yn}^2 + u^2}$. By replacing this asymptotic value of k_z into (7), the asymptotic behavior of $\tilde{G}_{mn}$ is obtained as

$$\tilde{G}_{mn}^a = \frac{1}{2\sqrt{k_{xm}^2 + k_{yn}^2 + u^2}}\left[e^{-\sqrt{k_{xm}^2+k_{yn}^2+u^2}(z-z')} + \sum_{i=1}^{N} A_i e^{-\sqrt{k_{xm}^2+k_{yn}^2+u^2}(z+z'-B_i)}\right]. \tag{10}$$

$\tilde{G}_{mn}^a$ is a smooth function that decays very slowly in the spectral domain. The property of slow convergence of G_p of

(1) is contained in this asymptotic $\tilde{G}^a_{mn}$. To overcome this problem, $\tilde{G}^a_{mn}$ is subtracted from and added to (1) yielding

$$G_p = \frac{1}{ab}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}(\tilde{G}_{mn} - \tilde{G}^a_{mn}) \cdot e^{-jk_{xm}(x-x')}\cdot e^{-jk_{yn}(y-y')} + \frac{1}{ab}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\tilde{G}^a_{mn}\cdot e^{-jk_{xm}(x-x')} \cdot e^{-jk_{yn}(y-y')} \quad (11)$$

The first series in (11) becomes now rapidly convergent because the asymptotic behavior of slow convergence is subtracted out.

C. The Improvement on the Convergence

The slow convergence of the asymptotic behavior, the second series in (11), is improved by converting it into the spatial domain. Here, the Poisson's summation formula is used:

$$\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} f(ma, nb) = \frac{1}{ab}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} F\left(\frac{2\pi m}{a}, \frac{2\pi n}{b}\right) \quad (12)$$

where $f(x,y)$ and $F(k_x,k_y)$ are a two-dimensional Fourier transform pair. They are related by the following equations:

$$F(k_x,k_y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-j(k_x x+k_y y)}dx\,dy \quad (13)$$

$$f(x,y) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(k_x,k_y)e^{+j(k_x x+k_y y)}dk_x\,dk_y \quad (14)$$

Therefore, in (11) the second spectral sum can be replaced using (12) by an equivalent spatial sum:

$$\frac{1}{ab}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\tilde{G}^a_{mn}\cdot e^{-jk_{xm}(x-x')}\cdot e^{-jk_{yn}(y-y')} = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} G^a_{mn}\cdot e^{jk^i_x ma}\cdot e^{jk^i_y nb} \quad (15)$$

where the spatial G^a_{mn} is the inverse Fourier transformation of the corresponding spectral $\tilde{G}^a_{mn}$. The exponential form (10) of the spectral $\tilde{G}^a_{mn}$, which is obtained using the complex image theory, allows the use of Sommerfeld identity [10] in performing the inverse Fourier operation analytically to obtain the following closed-form expression for G^a_{mn}:

$$G^a_{mn} = \frac{e^{-uR_0^{mn}}}{4\pi R_0^{mn}} + \sum_{i=1}^{N} A_i\frac{e^{-uR_i^{mn}}}{4\pi R_i^{mn}}, \qquad N \le 5 \quad (16)$$

where

$$R_0^{mn} = \sqrt{(x-x'-ma)^2+(y-y'-nb)^2+(z-z')^2}$$
$$R_i^{mn} = \sqrt{(x-x'-ma)^2+(y-y'-nb)^2+(z+z'-B_i)^2} \quad (17)$$

The spatial sum, the series on the right hand side of (15), expressed in terms of the closed-form expression (16), is rapidly convergent not only because of the small "N" but also because of the exponential decay of $\exp(-uR_0^{mn})$. Physically, G^a_{mn} represents the response at an observation point (x,y,z) due to a dipole point source located at z' and a set of "N" complex images having complex amplitudes (A_i's) and complex locations $(-z'+B_i)$. Both the point source and the set of complex images are located within the (mth, nth) unit cell and are radiating in a homogeneous medium with an imaginary wavenumber $k = -ju$. According to [3], the specific value of the attenuation constant u is determined numerically so that the spatial sum on the right hand side of (15) converges most rapidly.

Now, the final form of the periodic Green's function is obtained by substituting (15) in (11):

$$G_p = \frac{1}{ab}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}(\tilde{G}_{mn} - \tilde{G}^a_{mn}) \cdot e^{-jk_{xm}(x-x')}\cdot e^{-jk_{yn}(y-y')} + \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} G^a_{mn}\cdot e^{jk^i_x ma}\cdot e^{jk^i_y nb} \quad (18)$$

Therefore, the slowly convergent series of (1) has been successfully replaced by a combination of two rapidly convergent series as given in (18). The first series in (18) converges rapidly because two functions are being subtracted out that are asymptotically equal as m and n increase. The second series in (18) converges rapidly because the spatial functions involved decay exponentially as m and n increase.

III. Numerical Results

As a numerical example, we used the direct sum formula of (1) and the accelerated sum formula of (18) to evaluate the Green's function series of a two-dimensional (x,y) periodic array of $\tilde{x}$ -directed point dipole sources printed on a grounded substrate of thickness h and relative permittivity ϵ_r. The Green's function example being evaluated here is G_q which is the scalar potential of a point charge associated with a $\tilde{x}$-directed point dipole source. Its spectral-domain expression for the microstrip substrate problem is given in the Appendix.

Calculations of the periodic Green's function are made for the "on-plane" case where the original series of (1) has the slowest convergence, i.e., we choose $z = z' = 0$. Without any loss of generality, a source point is placed at $x' = y' = 0$ and the observation point is taken to be at different locations ρ where $\rho = \sqrt{x^2+y^2}$ with $x = y$, i.e., the observation point moves along the *diagonal* of a unit cell. Figs. 3 and 4 show the total number of terms in the accelerated formula (18) versus selected points of the normalized transverse distance ρ/λ_0. Here λ_0 is the free-space wavelength and ρ varies along the diagonal of a unit cell with dimensions $a = b = 1.1\lambda_0$. The results in these figures are obtained for different values of the substrate parameters ϵ_r and h.

The infinite spectral and spatial summations in (18) have been truncated when a convergence criterion e_c defined in [3] is satisfied. Here we choose $e_c = 5 \times 10^{-6}$, which is found

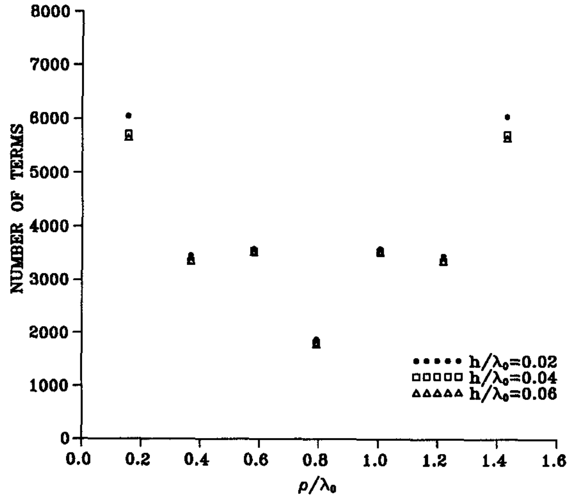

Fig. 3. Number of terms vs. the normalized transverse distance ρ/λ_0 for different values of the substrate thickness $h. (\epsilon_r = 2.55, f = 30$ GHz, $a = b = 1.1\lambda_0, k_x^i = k_y^i = 0)$.

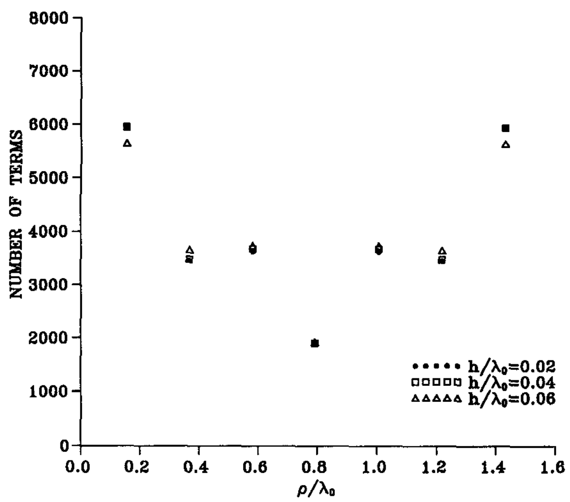

Fig. 4. Number of terms vs. the normalized transverse distance ρ/λ_0 for different values of the substrate thickness $h. (\epsilon_r = 9.8, f = 30$ GHz, $a = b = 1.1\lambda_0, k_x^i = k_y^i = 0$.

sufficient to ensure convergence. It is seen from Figs. 3 and 4 that the accelerated formula requires 78 × 78 terms at most to converge; whereas a straightforward summation of the unaccelerated series of (1) takes over *one million* (1000 × 1000) terms to satisfy the same value of the convergence criterion e_c. This clearly proves the significant reduction achieved in computation time by utilizing the accelerated formula (18) for the evaluation of the periodic Green's function.

In Fig. 5 we have plotted the magnitude of the periodic Green's function evaluated using the accelerated series of (18) versus the normalized transverse distance ρ/λ_0 for two different values of $\epsilon_r = 2.55$ and 9.8. Again the source point is located at $x' = y' = 0$ and the observation point is taken at different locations along the diagonal. The transverse distance ρ varies within the dimensions of two unit cells. It is obvious from this figure that the Green's function, which represents the charge scalar potential, is *symmetrical* within one unit cell as well as from one unit cell to another, i.e., around

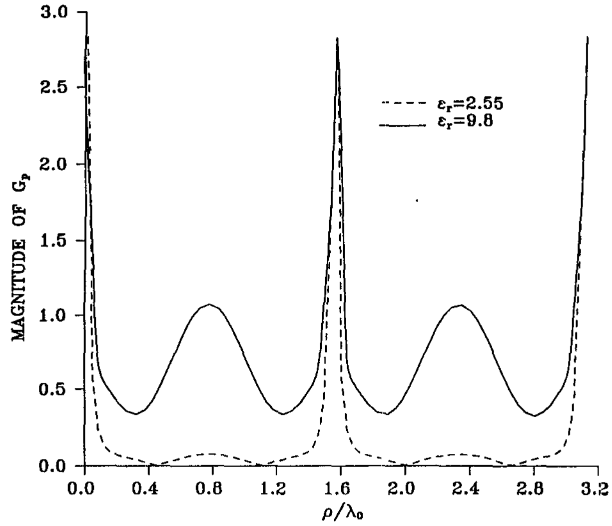

Fig. 5. Magnitude of the periodic Green's function G_p for the charge scalar potential calculated using the accelerated formula (18) vs. the normalized transverse distance ρ/λ_0. $(f = 30$ GHz, $a = b = 1.1\lambda_0, k_x^i = k_y^i = 0)$.

$\rho = 1.55 = \sqrt{2}(1.1\lambda_0)$. This indicates that the accelerated summation formula (18) is not only rapidly convergent but also accurate. It may be interesting to note that in both periodic sums of the accelerated and the unaccelerated series, the surface wave poles of the layered media are included in the spectral function $\tilde{G}_{mn}$ without extraction.

It is observed in Fig. 5, along the diagonal line ρ of the cell, the scalar potential periodic Green's function has a strong coloumb field at the sources: $\rho = 0, 1.56\lambda_0$ and $3.12\lambda_0$. This behavior is represented in the second sum of the spatial G_{mn}^a in (18). Between the sources, the Green's function is dominated by the surface wave represented by the first sum of the spectral $\tilde{G}_{mn}$ in (18). The surface wave increases in magnitude with increased dielectric ϵ_r.

Finally, it may be pointed out that the choice of the attenuation constant u has a more pronounced effect on the convergence of the spatial sum of G_{mn}^a and the spectral sum of $(\tilde{G}_{mn} - \tilde{G}_{mn}^a)$, but it has only a minor effect on the total of the two sums in (18). That is approximately: in varying u, an increase in convergence of the spatial sum, is compensated by the decrease in convergence of the spectral sum, and vice versa. The choice of a specific value for u therefore is not very critical. As a result, this paper chooses the value of $u = \pi/a$, similar to that in [3].

IV. Conclusions

This paper presented a new technique for the efficient computation of the layered media periodic Green's function. The technique uses the complex image theory and Poisson's summation formula to obtain a highly convergent combined *spectral-spatial* representation of the Green's function series. In comparison with other techniques which are applicable to the free-space periodic Green's function, our technique can be easily used for periodic Green's functions in general layered media. As indicated in [7], the generation of the complex images in this general multilayered case is not more difficult than the generation of complex images for the single layered microstrip example of the last section. This means that the developed technique is of potential importance as it can be applied to a wide class of problems where the modeling of periodic structures is required.

APPENDIX

A Green's function example used for $\tilde{G}_{mn}$ in (4) is the scalar potential of a point charge associated with a $\tilde{x}$ -directed dipole point source located above a microstrip substrate of thickness h and relative permittivity ϵ_r. The Green's function expression in the spectral domain is given by the following form of plane wave summations [11]:

$$\tilde{G}_q = \frac{1}{j2k_{z0}}[e^{-jk_{z0}(z-z')} + (R_{\rm TE} + R_q)e^{-jk_{z0}(z+z')}] \quad \text{(A1)}$$

where $R_{\rm TE}$ and R_q take into account the effects of the microstrip substrate and are given by

$$R_{\rm TE} = -\frac{r_{10}^{\rm TE} + e^{-j2k_{z1}h}}{1 + r_{10}^{\rm TE}e^{-j2k_{z1}h}} \quad \text{(A2)}$$

$$R_q = \frac{2k_{z0}^2(1-\epsilon_r)(1-e^{-j4k_{z0}h})}{(k_{z1}+k_{z0})(k_{z1}+\epsilon_r k_{z0})(1+r_{10}^{\rm TE}e^{-j2k_{z1}h})(1-r_{10}^{\rm TM}e^{-j2k_{z1}h})} \quad \text{(A3)}$$

$r_{10}^{\rm TE}$ and $r_{10}^{\rm TM}$ are simply the reflection coefficients of the TE and TM waves at the dielectric-air interface. They are given by

$$r_{10}^{\rm TE} = \frac{k_{z1} - k_{z0}}{k_{z1} + k_{z0}} \qquad r_{10}^{\rm TM} = \frac{k_{z1} - \epsilon_r k_{z0}}{k_{z1} + \epsilon_r k_{z0}} \quad \text{(A4)}$$

with

$$k_{z0}^2 + k_\rho^2 = k_0^2, \quad k_{z1}^2 + k_\rho^2 = \epsilon_r k_0^2, \quad \text{and} \quad k_\rho^2 = k_x^2 + k_y^2.$$

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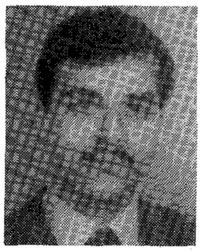

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